New models for reptile and amphibian removal data

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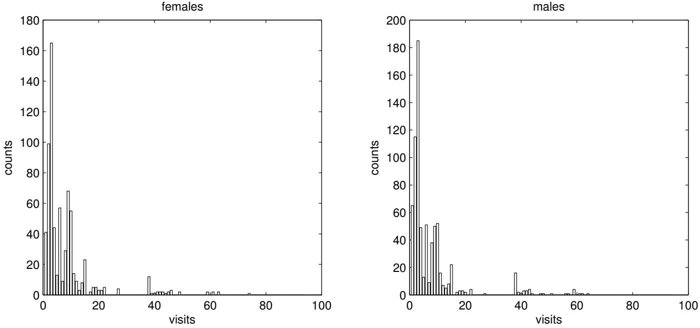
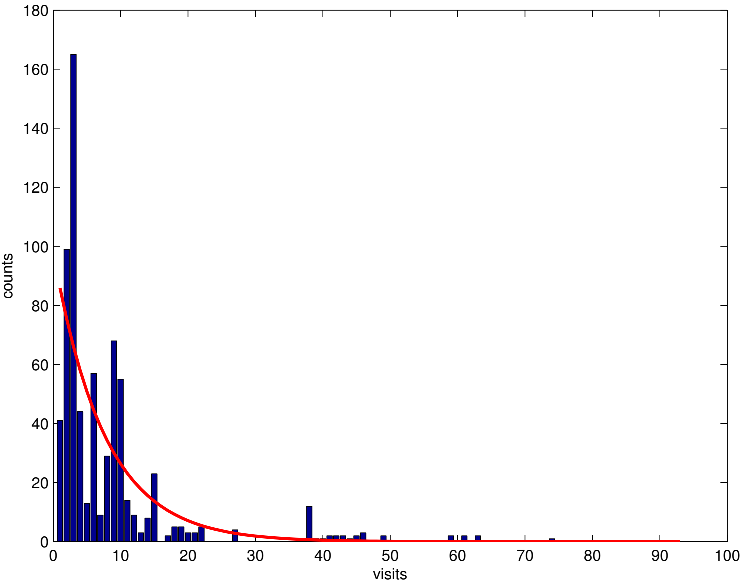
Outline

- Data
- Simple Removal model
- Covariates.
- Timed clearance
- Stopover models
- Discussion and recommendations.

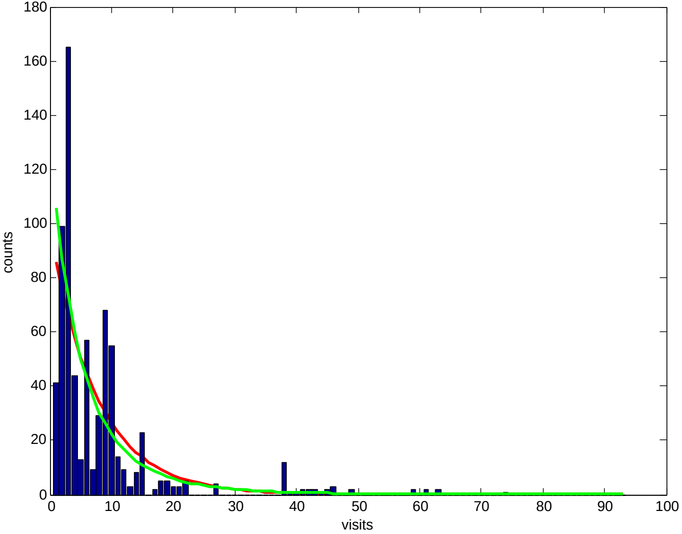
Basic Data: Great Crested Newts



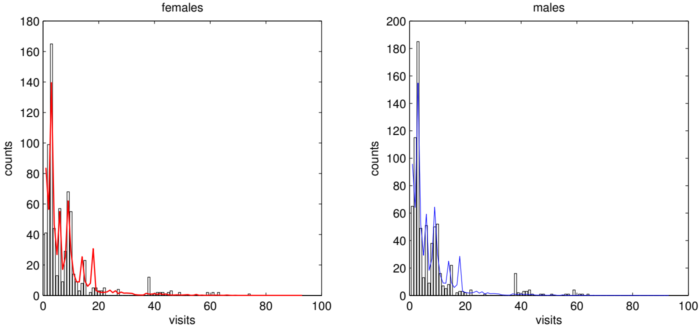
Result of fitting geometric model



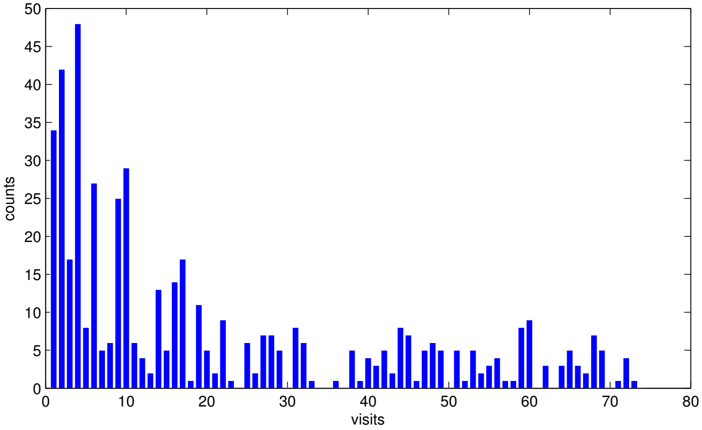
and then adding over dispersion

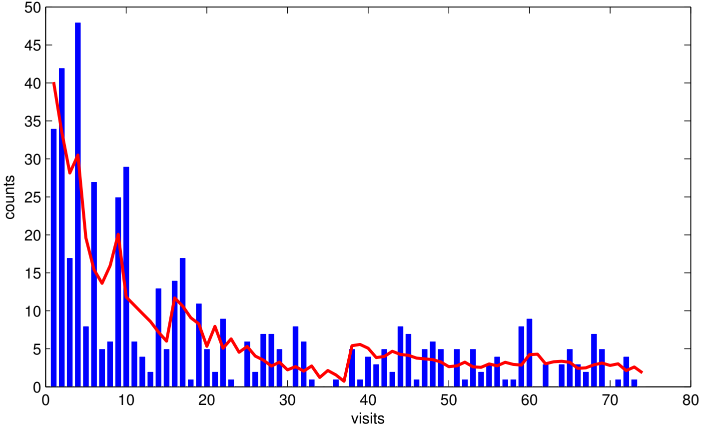


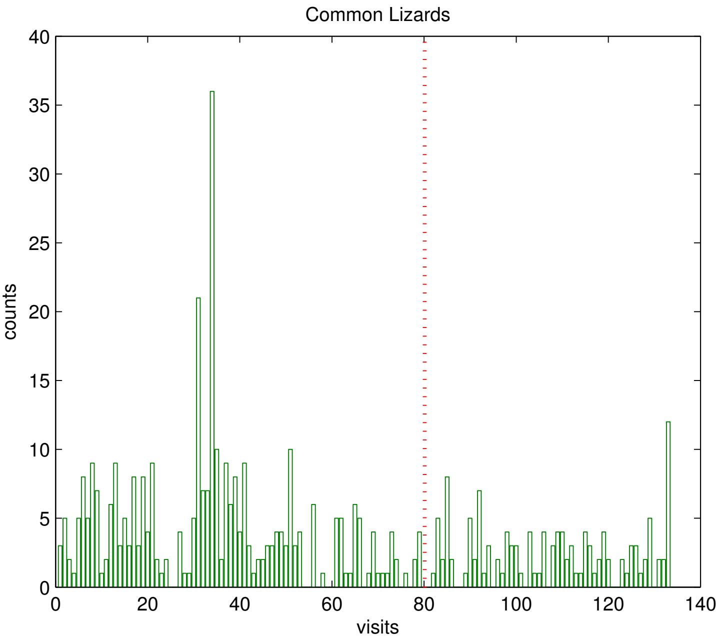
Geometric removal model with covariate

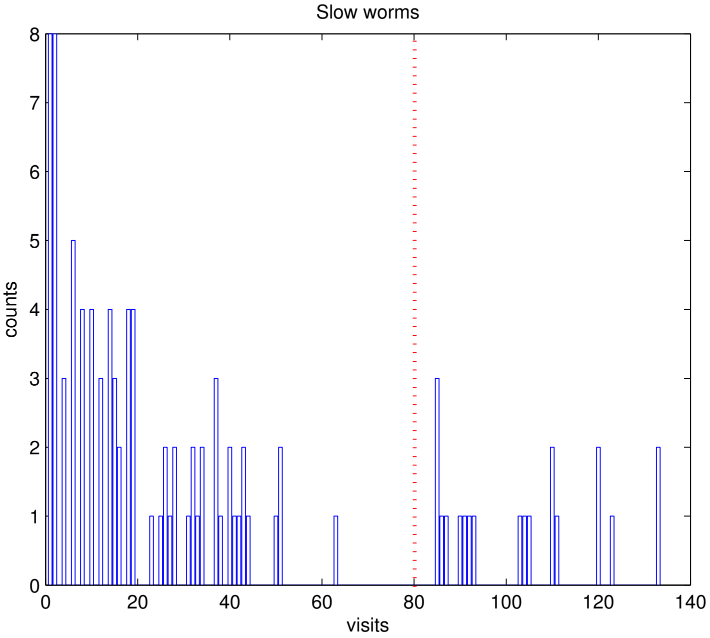


Timed clearance




counts
visits
0
5
10
15
20
25
30
35
40
45
50
0
10
20
30
40
50
60
70
80


Common Lizards
counts
visits
0
5
10
15
20
25
30
35
40
0
20
40
60
80
100
120
140


Slow worms
counts
visits
0
1
2
3
4
5
6
7
8
0
20
40
60
80
100
120
140

Modelling

- The simple removal model dates from papers by Moran (1951) and Zippin (1958)
- It assumes a constant capture probability, p.
- The other parameter is the desired population size, N.
- The same model applies to fecundability data, which record months to conception for human couples.
- 100 Smokers: 29 16 17 4 3 9 4 5 1 1 1 3 7; $\hat{N} = 95$.
- 486 Non-smokers: 198 107 55 38 18 22 7 9 5 3 6 6 12; $\hat{N} = 476$.

Three simple probability distributions

- Geometric: $\Pr(X = i) = (1-p)^{i-1}p$
- Multinomial:
  $\Pr(X_1 = x_1, X_2 = x_2, \ldots, X_T = x_t) = \frac{N!}{x_1!x_2!,\ldots,x_t!}\prod_{i=1}^{T} p_i^{x_i}$
- Beta-geometric:
  Here we give p a Beta distribution.

Notation and likelihood

- N: initial population size
- n_k: size of the k^{th} sample removed from the population, $k = 1, 2, \ldots, T$
- $x_k = \sum_{j=1}^{k-1} n_j$, $k = 2, 3, \ldots, T+1; x_1 = 0$.

For example: $\mathbf{n} = (65, 115, 185, \ldots)$

$$\mathbf{x} = (0, 65, 180, 365, \ldots).$$

We then form the likelihood:

$$\mathrm{L}(N, p; \mathbf{n}) = \frac{N!}{(\prod_{k=1}^{T} n_k!)(N - x_{T+1})!} \left[\prod_{k=1}^{T} \{p(1-p)^{k-1}\}^{n_k} \right] (1-p)^{T(N - x_{T+1})}$$

Maximum-likelihood estimates

The likelihood simplifies to give

$$\mathrm{L}(N, p; \mathbf{n}) = \frac{N!}{(\prod_{k=1}^{T} n_k!)(N - x_{T+1})!} p^{x_{T+1}} (1-p)^{TN - \sum_{k=1}^{T+1} x_k}$$

Maximum-likelihood estimates of the two parameters are given by the solutions to the equations:

$$\hat{N} = \frac{x_{T+1}}{1-(1-\hat{p})^T}$$

$$\frac{\hat{p}}{1-\hat{p}} - \frac{T(1-\hat{p})^T}{1-(1-\hat{p})^T} = \frac{\sum_{k=1}^{T}(k-1)n_k}{x_{T+1}}$$

Covariates

- In practice we can expect the capture probability to vary
- for fecundability, conception might vary with age and parity
- for animals capture might vary with temperature. For GCN, minimum air temperature was used
- We may have a logistic transformation, for a covariate w:

$$p = \frac{1}{1 + \exp(\alpha + \beta w)}.$$

- It is often necessary to choose the best covariate(s) from a relevant set.
- Overdispersion may also be included, eg., using a beta-geometric distribution.

Model for timed clearences

- For the timed clearance data, at fixed times parts of the study area were cleared.
- This was modelled by assuming a global population of animals of size N, to be estimated.
- Fractions of this number were assumed to be available for capture during each time interval. Cf Stop-over modelling.
- Using maximum likelihood, the fractions were estimated as: 0.45, 0.10, 0.17, 0.28, 0.00.
- The estimated number of animals not observed was $\hat{N}_0 = 51$.

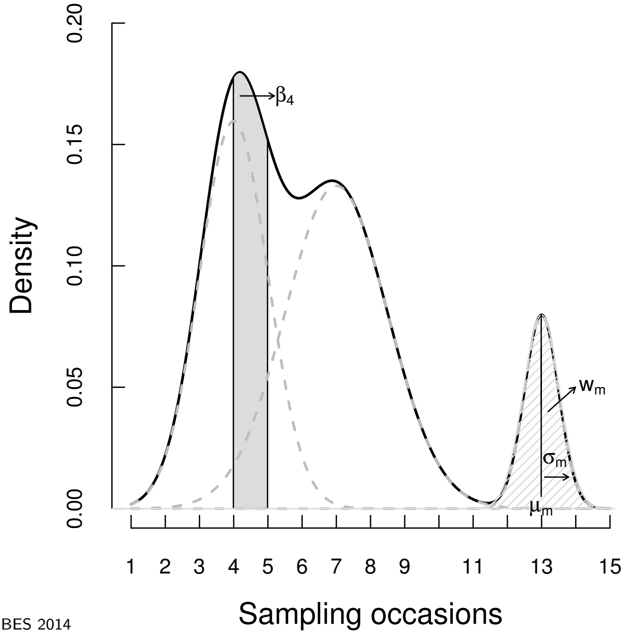
Stop-over model parameters

- N: population size; M: number of arrival groups.
- w_m, μ_m and σ_m, $m = 1, \ldots, M$: The population fractions, mean arrival times and standard deviations of arrival times of the M arrival groups, $\sum_{m=1}^{M} w_m = 1$. The population fraction that arrived between occasions $b-1$ and b is denoted by β_{b-1}. In terms of the mixture components,

$$\beta_{b-1} = \sum_{m=1}^{M} w_m \left\{F_m(b) - F_m(b-1)\right\}, \; b = 2, \ldots, T-1$$

where $F_m(b) = P(X \leq b)$ when $X \sim N(\mu_m, \sigma_m^2)$. The first and last intervals are open-ended with
$\beta_0 = \sum_{m=1}^{M} w_m F_m(1)$ and
$\beta_{T-1} = 1 - \sum_{m=1}^{M} w_m F_m(T-1)$, $\forall m$, ensuring that the entry parameters sum to 1 i.e. $\sum_{b=1}^{T} \beta_{b-1} = 1$.

Stop-over model: $M = 3$



Forming the likelihood

If an individual belongs to category t, $t = 1, \ldots, T$, then it was removed on sampling occasion t. The unknown number of individuals that were never detected and therefore never removed belong to category $T + 1$.

The probability of belonging to category t, γ_t, is:

$$\gamma_t = \begin{cases} \sum_{b=1}^{t} \left[\beta_{b-1} \left\{\prod_{k=b}^{t-1}(1-p_k)\right\}\right] p_t, & t = 1, \ldots, T \\ 1 - \sum_{t=1}^{T} \gamma_t = \sum_{b=1}^{T} \left[\beta_{b-1} \prod_{k=b}^{T}(1-p_k)\right], & t = T+1 \end{cases},$$

The likelihood is multinomial with $T + 1$ cells, γ_t, $t = 1, \ldots, T + 1$ probabilities and n_t, $t = 1, \ldots, T + 1$ frequencies.

Stop-over likelihood

The parameters are given by

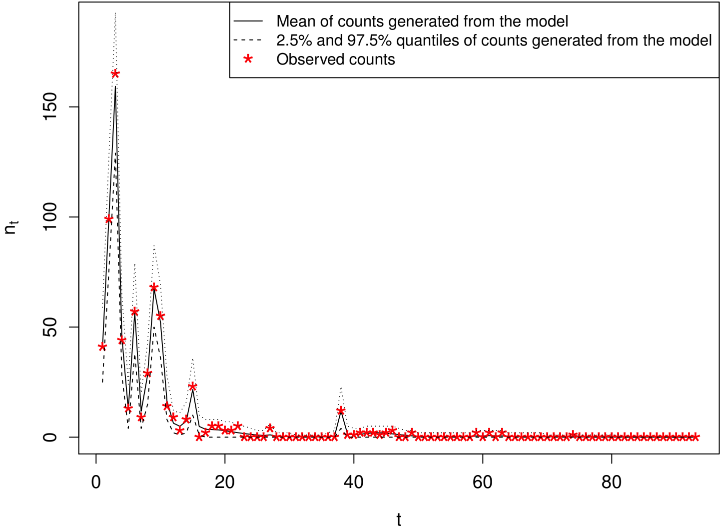
$$\boldsymbol{\theta} = (M, N, (w_m, \mu_m, \sigma_m)_{m=1,\dots,M}, (p_t)_{t=1\dots,T})$$

and the likelihood is:

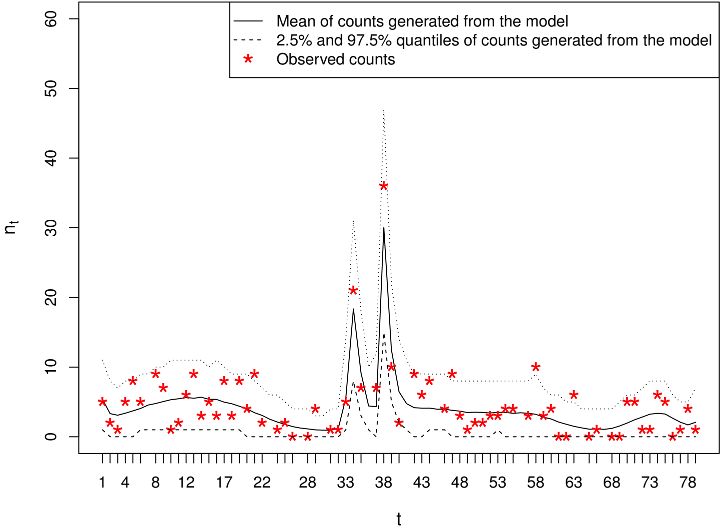
$$p(\mathbf{y}|\boldsymbol{\theta}) = \frac{N!}{\left(\prod_{t=1}^{T} n_t!\right)(N-D)!}\left\{\prod_{t=1}^{T} \gamma_t^{n_t}\right\}\gamma_{T+1}^{N-D},$$

where $D = \sum_{t=1}^{T} n_t$. We assume constant capture in the applications.

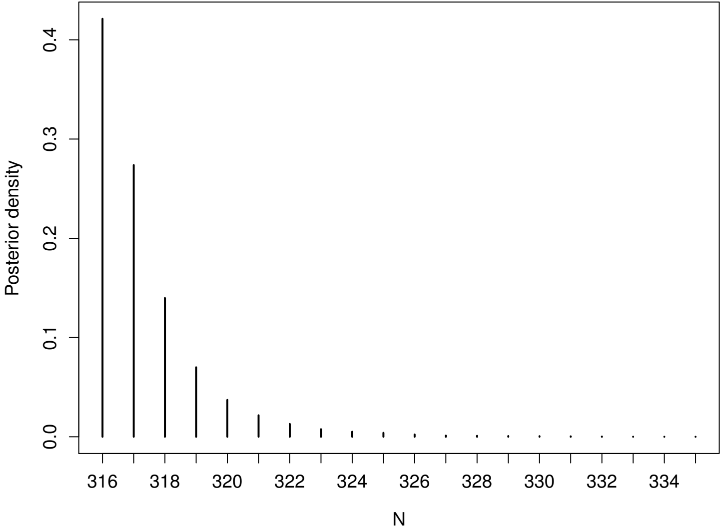
Stop-over, RJMCMC, GCN



Stop-over, RJMCMC, lizard



Posterior distribution for N



References

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3. Moran (1951) *Biometrika*, **38**, 307–311.
4. Ridout and Morgan (1991) *Biometrics*, **47**, 1423–1433.
5. Zippin (1958) *J. Wildlife Management*, **22**, 82–90.